

A simple generalization of Geršgorin theorem

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Abstract

According to the famous Geršgorin theorem, all eigenvalues of a given $n \times n$ matrix A lie in the union of n Geršgorin disks $\Gamma_i(A)$, and, at the same time, in the union of n Geršgorin disks $\Gamma_i(A^T)$ for transpose of A . But, if we take the intersection of $\Gamma_i(A)$ and $\Gamma_i(A^T)$ for each index i , since the centers of $\Gamma_i(A)$ and $\Gamma_i(A^T)$ are the same, we will get another n circles, the union of which does not necessarily contain all eigenvalues of the matrix A . What should be added to this union, in order to obtain a proper eigenvalue inclusion set? Surprisingly, the answer can be obtained by using one special subclass of H-matrices, introduced long time ago by Ostrowski.

Keywords

Eigenvalue localization, Geršgorin theorem.

References

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