Extension of models with orthogonal block structure

João T. Mexia, Paulo C. Rodrigues, and Miguel Fonseca

Nova University of Lisbon, Portugal

Abstract

TA new approach for the treatment of unbalanced mixed models is presented. Given a mixed model $\mathbf{Y}^0$ its extensions are the models $\mathbf{Y} = \mathbf{L}\mathbf{Y}^0 + \mathbf{e}$ where $\mathbf{L}$ is a matrix with linearly independent column vectors and $\mathbf{e}$ is an error vector independent from $\mathbf{Y}^0$ The study is centered on core models with orthogonal block structure. Crossing and nesting of core models is carried out as well as extensions obtained using matrices $\mathbf{L}$ such that $\mathbf{L}^+\mathbf{L} = \mathbf{I}$. These last extensions preserve balance in the core models.

This approach can be seen as an extension of the method for balanced random models with unequal frequencies in the last stage (Khuri and Ghosh), allowing, for instance, replicates to follow a linear correlation structure with exogenous covariates within each cell, and the use of mixed core models.

Keywords

Commutative Jordan algebras, Core model, Mixed models, Orthogonal block structure, Orthogonal models.

References

Bailey, R.A. (1991). Strata for randomized experiments. *J. Roy. Statist. Soc. Ser. B 53*, 27–78.

Bailey, R. A. (2007). Optimal nested row-column designs with specified components. *Biometrika 94*, 459–468.

Brien, C.J. and R.A. Bailey (2006). Multiple randomizations. *J. Roy. Statist. Soc. Ser. B 68*, 571–599.

Caliński, T. and S. Kageyama (2000). *Block designs: a randomization approach. Vol. I: Analysis.* Lecture Notes in Statistics, Springer-Verlag, New York.

Caliński, T. and S. Kageyama (2003). *Block designs: a randomization approach. Vol. II: Design.* Lecture Notes in Statistics, Springer-Verlag, New York.

Covas, R. (2007). *Orthogonal Mixed Models and Commutative Jordan Algebras.* Ph.D Thesis, Nova University of Lisbon, Portugal.

Fayyad, R., F.A. Graybill, and R.K. Burdick (1996). A note on exact tests for variance components in unbalanced random and mixed linear models. *Biometrics 51*, 306–308.

Fonseca, M., J.T. Mexia, and R. Zmyślony (2006). Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra Appl. 417*, 75–86.

Fonseca, M., J.T. Mexia, and R. Zmyślony (2008). Inference in normal models with commutative block orthogonal structure. *Acta et Commentationes Universitatis Tartuensis de Mathematica* (in press).

Gelman, A. (2005). Analysis of variance - Why it is more important than ever. *Ann. Statist. 33*, 1–53.

Houtman, A.M. and T.P. Speed (1983). Balance in designed experiments with orthogonal block structure. *Ann. Statist. 11*, 1069–1983.

Jordan, P., J. Von Neumann, and E. Wigner (1934). On an algebraic generalization of the quantum mechanical formalism. *Ann. of Math. 35*, 29–64.

Khuri, A.I. and M. Ghosh (1990). Minimal sufficient statistics for the unbalanced two-fold nested model. *Stat. Probab. Lett. 10*, 351-353.

Khuri, A.I., T. Mathew, and B.K. Sinha (1997). *Statistical tests for mixed linear models.* John Wiley & Sons, New York.

Lehmann, E.L. and G. Casella (1988). *Theory of point estimation.* Second ed., Springer Texts in Statistics, Springer-Verlag, New York.

Liu, X.-Q., J.-Y. Rong, and X.-Y Liu (2008). Best linear unbiased prediction for linear combinations in general mixed linear models. *J. Multivariate Anal. 99*, 1503-1517.

Malley, J.D. (1994). *Statistical applications of Jordan algebras.* Lecture Notes in Statistics, Springer-Verlag, New York.

Nelder, J.A. (1965). The analysis of randomized experiments with orthogonal block structure. I. Block structure and the null analysis of variance. *Proc. R. Soc. Lond. Ser. A 283*, 147-162.

Nelder, J.A. (1965). The analysis of randomized experiments with orthogonal block structure. II. Treatment structure and the general analysis of variance. *Proc. R. Soc. Lond. Ser. A 283*, 163-178.

Ofversten, J. (1993). Exact Tests for Variance Components in Unbalanced Mixed Linear Models. *Biometrics 49*, 45–57.

Rao, C.R. and M.B. Rao (1998). *Matrix algebra and its applications to statistics and econometrics.* World Scientific Publishing, Singapore.

Schaalje, B., J. Zhang, S.G. Pantula, and K.H. Pollock (1991). Analysis of Repeated-Measurements Data from Randomized Block Experiments. *Biometrics 47*, 813–824.

J.R. Schott (2005). *Matrix Analysis For Statistics.* Second ed., John Willey & Sons, New York.

Seely, J. (1970). Linear spaces and unbiased estimation. *Ann. Math. Statist. 41*, 1725–173.

Seely, J. (1970). Linear spaces and unbiased estimation - application to the mixed linear model. *Ann. Math. Statist. 41*, 1735–1748.

Seely, J. (1971). Quadratic subspaces and completeness, *Ann. Math. Statist. 42*, 710–721.

Seely, J. and G. Zyskind (1971). Linear spaces and minimum variance unbiased estimation. *Ann. Math. Statist. 42*, 691–703.

Steeb, W.-H. (1991). *Kronecker product of matrices and applications.* Bibliographisches Institut, Mannheim, 1991.

Van Leeeuwen, D., J. Seely, and D. Birkes (1998). Sufficient conditions for orthogonal designs in mixed linear models. *J. Statist. Plann. Inference 73*, 373–389.

Van Leeeuwen, D., D. Birkes, and J. Seely (1999). Balance and orthogonality in designs for mixed classification models. *Ann. Statist. 27*, 1927–1947.

Zmyślony, R. (1978). A characterization of best linear unbiased estimators in the general linear model, in: Mathematical statistics and probability theory, *Proc. Sixth Internat. Conf.*, Wisła, 1978, Lecture Notes in Statist., 2, Springer, New York-Berlin, pp. 365–373.

Zmyślony, R. and H. Drygas (1992). Jordan algebras and Bayesian quadratic estimation of variance components. *Linear Algebra Appl. 168*, 259–275.