Interpolation-based model reduction

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Abstract

The behavior of processes in electrical networks, mechanics, aeronautics, civil engineering, micro-electro-mechanical-systems, weather prediction and many others can often be mathematically modeled by dynamical systems. Such models are usually determined by systems of partial differential equations. Their linearization and discretization by means of finite element or finite difference methods leads to high dimensional systems of linear ordinary differential or difference equations. The number of resulting equations depends on the quality of discretization and is typically very large. It can easily reach a few millions.

It is then desirable to find reduced order systems of the same form but of lower complexity, whose input-output behavior approximates the behavior of the original system. This input-output behavior is fully described by the transfer function of the system. Recently introduced tangential interpolation techniques for the transfer function can lead to numerically efficient methods for computing reduced order approximants of very large systems. A good choice of interpolation points and tangential directions remains then as a problem. Current methods are designed for approximating asymptotically stable systems. However, some processes such as weather development lead to unstable linear time invariant (LTI) systems.

In this talk, model reduction techniques for discrete-time LTI dynamical systems shall be presented. We will focus here on the problem of finding a reduced order system which minimizes the approximation error between original and reduced order models in the $h_{2,\alpha}$-norm. For this optimization problem the first order necessary conditions will be presented. From these conditions we will derive a good choice of interpolation data. We will prove that this method is suitable for unstable systems. Based on the approach introduced here we will suggest how other existing model reduction techniques can be applied to unstable systems. We will also show the relation between optimal model reduction in the $h_{2,\alpha}$-norm and (tangential) rational Hermite interpolation. On the basis of the established theory, we shall propose an iterative rational interpolation algorithm which, if it converges, provides a reduced-order system that satisfies the first order necessary $h_{2,\alpha}$-optimality conditions. The benefit of the new approach to model order reduction of unstable systems shall also be demonstrated in several numerical experiments with shallow water test models.

For detailed study of model reduction techniques presented in this talk we refer to Kubalińska (2008) and references therein.

References

Kubalińska, D. (2008). *Optimal interpolation-based model reduction.* PhD Thesis, Universität Bremen.