

The matrix negative Pell equation and the application of some spectral properties of matrices to number theory

Aleksander Grytczuk and Izabela Kurzydło

University of Zielona Góra, Poland

Abstract

We give necessary and sufficient conditions for solvability of the matrix negative Pell equation $X^2 - dY^2 = -I$, where $d \in N$. We consider the generalization of this equation: $\sum_{i=1}^n X_i^2 - d \sum_{i=1}^n Y_i^2 = -I$, where $d \in N$.

In the second part of this lecture we present the application of some spectral properties of matrices to the solution to the Schinzel - Zassenhaus conjecture.

Keywords

Matrix negative Pell equation, Some spectral properties of matrices.

References

- Domiaty, R.Z. (1966). Solutions of $x^4 + y^4 = z^4$ in 2×2 integral matrices. *Amer. Math. Monthly* 73, 631.
- Grytczuk, A. (1998). On a conjecture about the equation $A^{mx} + A^{my} = A^{mz}$. *Acta Acad. Paed. Agrieusis, Sectio Math.* 25, 61–70.
- Grytczuk, A. and I. Kurzydło. The necessary and sufficient condition for the solvability of Diophantine matrix equation $A^{mx} + A^{my} + A^{mz} = A^{mw}$. Submitted.
- Grytczuk, A. and I. Kurzydło. Note on the Schinzel - Zassenhaus conjecture. Submitted.
- Grytczuk, A. and M. Szałkowski (1991). Spectral properties of some matrices. *Acta Acad. Paed. Agriensis Sectio Math.* 20, 43–50.
- Khazanov, A. (1995). Fermat's equation in matrices. *Serdica Math. J.* 21, 19–40.
- Le, M. and C. Li (1995). On Fermat's equation in integral 2×2 matrices. *Period. Math. Hung.* 31, 219–222.
- Patay, Z. and A. Szakacs (2002). On Fermat's problem in matrix rings and groups. *Publ. Math. Debrecen* 61/3-4, 487–494.
- Schinzel, A. and H. Zassenhaus (1965). A refinement of two theorems of Kronecker. *Michigan Math. J.* 12, 81–85.