

# The numerical range of non square matrices

Aikaterini Aretaki and John Maroulas

*National Technical University of Athens, Greece*

## Abstract

TA presentation of numerical range for rectangular matrices is undertaken in this paper, introducing two different definitions and elaborating basic properties. Then we are extended to the treatment of rank-k numerical range.

## Keywords

Numerical range, Projectors, Matrix norms, Singular values.

## References

- Bonsall, F.F. and J. Duncan (1971). *Numerical Ranges of Operators on Normed Spaces and of Elements of Normed Algebras*. London Mathematical Society Lecture Note Series, Cambridge University Press, New York.
- Bonsall, F.F. and J. Duncan (1973). *Numerical Ranges II*. London Mathematical Society Lecture Notes Series, Cambridge University Press, New York.
- Choi, M.D., M. Giesinger, J.A. Holbrook, and D.W. Kribs (2008). Geometry of higher-rank numerical ranges. *Linear Multilinear Algebra* 56(1), 53–64.
- Choi, M.D., J.A. Holbrook, D.W. Kribs, and K. Zyczkowski. Higher-rank numerical ranges of unitary and normal matrices. *Preprint*: <http://arxiv.org/quant-ph/0608244>.
- Choi, M.D., D.W. Kribs, and K. Zyczkowski (2006). Quantum error correcting codes from the compression formalism. *Reports on Mathematical Physics* 58, 77–86.
- Choi, M.D., D.W. Kribs, and K. Zyczkowski (2006). Higher-rank numerical ranges and compression problems. *Linear Algebra Appl.* 418, 828–839.
- Chorianopoulos, Ch., S. Karanasios, and P. Psarrakos (2009). A definition of numerical range of rectangular matrices. *Linear Multilinear Algebra*. To appear.
- Gustafson, K.E. and D.K.M. Rao (1997). *Numerical Range. The Field of Values of Linear Operators and Matrices*. Springer-Verlag, New York.
- Halmos, P.R. (1982). *A Hilbert Space Problem Book*, 2nd Ed. Springer-Verlag, New York.
- Horn, R.A. and C.R. Johnson (1985). *Matrix Analysis*. Cambridge University Press, Cambridge.

- Horn, R.A. and C.R. Johnson (1991). *Topics in Matrix Analysis*. Cambridge University Press, Cambridge.
- Lancaster, P. and L. Rodman (1995). *Algebraic Riccati Equations*. Oxford Science Publications, The Clarendon Press Oxford University Press, New York.
- Li, C.K., Y.T. Poon, and N.S. Sze. Condition for the higher rank numerical range to be non-empty. *Linear Multilinear Algebra*. To appear.
- Li, C.K. and N.S. Sze (2008). Canonical forms, higher rank numerical ranges, totally isotropic subspaces, and matrix equations. *Proceedings of the American Mathematical Society* 136, 3013–3023.
- Thompson, R.C. (1972). Principal submatrices IX: Interlacing inequalities for singular values of submatrices. *Linear Algebra Appl.* 5, 1–12.
- Woerdeman, H.J. (2007). The higher rank numerical range is convex. *Linear and Multilinear Algebra* 56(1), 65–67.