On MLEs in a multivariate linear model

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Abstract

Consider the following multivariate linear model

$$\mathbf{Y} = \mathbf{A}_1\mathbf{B}_1\mathbf{C}_1 + \mathbf{A}_2\mathbf{B}_2\mathbf{C}_2 + \mathbf{A}_3\mathbf{B}_3\mathbf{C}_3 + \mathbf{E}, \tag{1}$$

where the matrices $\mathbf{A}_i$ and $\mathbf{C}_i \in \mathbb{R}^{q_i \times p}$, $i = 1, 2, 3$, are known. The matrix $\mathbf{Y} \in \mathbb{R}^{n \times p}$ is an observations matrix and $\mathbf{B}_i \in \mathbb{R}^{m_i \times q_i}$, $i = 1, 2, 3$, are matrices of unknown parameters. The matrix $\mathbf{E} \in \mathbb{R}^{n \times p}$ is a matrix of random errors, normally distributed, with expectation $E[\mathbf{E}] = \mathbf{0}$ and with dispersion matrix $D[\mathbf{E}] = D[\text{vec}(\mathbf{E})] = \mathbf{\Sigma} \otimes \mathbf{I}_n$, where $\mathbf{\Sigma} \in \mathbb{R}^p_>$ is an unknown positive definite matrix, $\text{vec}(\mathbf{E})$ denotes the vector in $\mathbb{R}^{pn}$ formed by putting the columns of $\mathbf{E} \in \mathbb{R}^{n \times p}$ under each other, starting from the left, and $\otimes$ denotes the Kronecker product. The matrices $\mathbf{A}_i$ (between individuals design matrices) are used to design the experiment, i.e. lay out treatments in an appropriate way, whereas the $\mathbf{C}_i$ matrices (within individuals design matrices) are used to describe the relation between the response variables.

The model is a generalized version of the Growth Curve model (Potthoff & Roy, 1964) and is sometimes termed sums of profiles model (see Verbyla & Venables, 1988). The model may be viewed as a multivariate seemingly unrelated regression (SUR) model. However to obtain explicit maximum likelihood estimators a nested subspace condition has to be imposed. This can be performed in two different ways leading to different parameterizations. However, it is only for one of them where a lot of detailed knowledge such as uniqueness conditions for MLEs, moments and asymptotics has been presented (e.g. see Kollo & von Rosen, 2005; Chapter 4). When for example discussing Kiefer optimality (see Filipiak et al., 2008), unfortunately, we need results for the estimators of parameters in the other parametrization.

In the paper we are going to refer to the two models as Model I and Model II: ($\mathcal{R}(\bullet)$ denotes the column space)

Definition 1. *Let all matrices be the same as in (1). Model I:*

$$\mathbf{Y} = \mathbf{A}_1\mathbf{B}_1\mathbf{C}_1 + \mathbf{A}_2\mathbf{B}_2\mathbf{C}_2 + \mathbf{A}_3\mathbf{B}_3\mathbf{C}_3 + \mathbf{E}, \qquad \mathcal{R}(\mathbf{C}_3') \subseteq \mathcal{R}(\mathbf{C}_2') \subseteq \mathcal{R}(\mathbf{C}_1');$$

Model II:

$$\mathbf{Y} = \mathbf{A}_1\mathbf{B}_1\mathbf{C}_1 + \mathbf{A}_2\mathbf{B}_2\mathbf{C}_2 + \mathbf{A}_3\mathbf{B}_3\mathbf{C}_3 + \mathbf{E}, \qquad \mathcal{R}(\mathbf{A}_3) \subseteq \mathcal{R}(\mathbf{A}_2) \subseteq \mathcal{R}(\mathbf{A}_1).$$

The aim of this paper is to present results in parallel for both Model I and Model II and in particular derive new results for Model I. Estimators of the unknown parameters are presented as well as moments of these estimators. Conditions for uniqueness of the estimators will also be given.

Keywords

Extended growth curve model, MLEs, Moments.

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