Extrapolated alternating projection methods

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Abstract

Let A and B be nonempty, closed and convex subsets of the Euclidean space $\mathbb{R}^n$ and let $T = P_A P_B : A \to A$ be the alternating projection operator with nonempty $\operatorname{Fix} T$. It is well known that sequences generated by the von Neumann alternating projection method $x_{k+1} = Tx_k$ converge to a fixed point of the operator T. We deal with generalizations of the von Neumann method, which have the form $x_{k+1} = P_A\,(x_k + \lambda_k\,\sigma\,(x_k)\,(Tx_k - x_k))$, where $\lambda_k \in [0, 2]$ is a relaxation parameter and $\sigma : A \to (0, +\infty)$ is a stepsize function. We propose new step sizes $\sigma_k = \sigma(x_k)$ which generalize the previous modifications of the von Neumann method (see Bauschke et al., 2006; Bauschke et al., 2003; Combettes, 1994; Gurin et al., 1967). We give sufficient conditions for the convergence to a fixed point of T of sequences generated by the proposed methods. We also present the results of preliminary numerical experiments.

Keywords

Alternating projection operator, Stepsize function, Convergence.

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